COLEGIO ALMA'S
bilingual school

APELLIDOS Y NOMBRE: Corrección Examen Evaluación

CURSO: 2º Bachillerato Nº 2ª Evaluación

FECHA: 02-03-2018 ASIGNATURA: Matemáticas 2.

1.-

$$A-mI=\begin{pmatrix}-m & 1 & 1\\ 0 & 3-m & 0\\ 0 & -1 & 3-m\end{pmatrix}$$

a) $\begin{vmatrix}-m & 1 & 1\\ 0 & 3-m & 0\\ 0 & -1 & 3-m\end{vmatrix} = -m(m-3)^2=0 < \begin{matrix} m=0 \\ m=3 \end{matrix}$ Si $m\in\mathbb{R}-\{0,3\} \Rightarrow |A-mI|\neq 0 \Rightarrow \exists (A-mI)^{-1}$

b) Para $m=2 \rightarrow |A-2I| = -2(-1)^2 = -2$

$$A-2I=\begin{pmatrix}-2 & 1 & 1\\ 0 & 1 & 0\\ 0 & -1 & 1\end{pmatrix} \rightarrow Adj(A-2I)=\begin{pmatrix}1 & 0 & 0\\ -2 & -2 & -2\\ -1 & 0 & -2\end{pmatrix} \rightarrow Adj(A-2I)^t=\begin{pmatrix}1 & -2 & -1\\ 0 & -2 & 0\\ 0 & -2 & -2\end{pmatrix}$$

$$(A-2I)^{-1}=\begin{pmatrix}-1/2 & 1 & 1/2\\ 0 & 1 & 0\\ 0 & 1 & 1\end{pmatrix}$$

2.-

$$\left.\begin{matrix}\pi_1 \equiv 3x+y+2z-1=0\\ \pi_2\equiv 2x-y+3z-1=0\end{matrix}\right\} \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 1 & 2\\ 2 & -1 & 3\end{vmatrix} = 5\vec{i}-5\vec{j}-5\vec{k} \rightarrow \vec{u}(1,-1,-1)$$

$$\left.\begin{matrix}x=1-2t\\ y=-1+t\\ z=1+t\end{matrix}\right\} \begin{matrix}P(1,-1,1)\\ \vec{v}(-2,1,1)\end{matrix} \qquad \begin{vmatrix}x-1 & y+1 & z-1\\ 1 & -1 & -1\\ -2 & 1 & 1\end{vmatrix}=0$$

$$1(y+1)-(z-1)=0 \rightarrow \boxed{y-z+2=0}$$

3.- <u>Matriz Asociada</u> <u>Matriz Ampliada</u>

$$A=\begin{pmatrix}1 & 1 & 1\\ m & 1 & m+1\\ 1 & m & m\end{pmatrix} \qquad A^*=\begin{pmatrix}1 & 1 & 1 & 1\\ m & 1 & m+1 & 1\\ 1 & m & m & m+2\end{pmatrix}$$

$$\begin{vmatrix}1 & 1 & 1\\ m & 1 & m+1\\ 1 & m & m\end{vmatrix} = (m+m^2+m+1)-(1+m^2+m+m^2) = -m^2+m=0 < \begin{matrix} m=0\\ m=1\end{matrix}$$

Si $m\in\mathbb{R}-\{0,1\} \Rightarrow rang A = rang A^* = 3 \Rightarrow$ Sistema Compatible Determinado

COLEGIO ALMA'S
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APELLIDOS Y NOMBRE:...

CURSO: .... **Nº** ....

FECHA:............................ **ASIGNATURA:** ....

Si $m=1 \Rightarrow$ rang $A^* = 3 \neq$ rang $A = 2 \Rightarrow$ Sistema Incompatible

$$A^* = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 3 \end{pmatrix} \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{vmatrix} = 2 \neq 0$$

Si $m=0 \Rightarrow$ rang $A^* = 3 \neq$ rang $A = 2 \Rightarrow$ Sistema Incompatible

$$A^* = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 2 \end{pmatrix} \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = (2+0+1)-(1+0+0) = 3-1 = 2 \neq 0$$

Para $m=-1 \Rightarrow |A| = -2$

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & -1 \end{vmatrix}}{-2} = \frac{-2}{-2} = \boxed{1}$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & -1 \end{vmatrix}}{-2} = \frac{-4}{-2} = \boxed{2} \qquad z = \frac{\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & -1 & -1 \end{vmatrix}}{-2} = \frac{4}{-2} = \boxed{-2}$$

4.- $\left.\begin{matrix} \pi_1: x+y=0 \\ \pi_2: x=0 \\ B(-1,1,1) \end{matrix}\right\}$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -\vec{k} \rightarrow \left.\begin{matrix} \vec{v}(0,0,1) \\ B(-1,1,1) \end{matrix}\right\}$ $\dfrac{x+1}{0} = \dfrac{y-1}{0} = \dfrac{z}{1}$

a) $\left.\begin{matrix} x+1=0 \\ y-1=0 \end{matrix}\right\}$ b) $\alpha = \arccos \dfrac{1}{\sqrt{2}} = \boxed{45^\circ}$

5.- $XB = A \rightarrow X = AB^{-1}$

$$\begin{vmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = (0+1+1)-(0+1-1) = 2 \neq 0 \Rightarrow \exists B^{-1}$$

$$\text{Adj}B = \begin{pmatrix} -1 & +2 & -1 \\ -2 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \rightarrow \text{Adj}B^t = \begin{pmatrix} -1 & -2 & 1 \\ 2 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix} \rightarrow B^{-1} = \begin{pmatrix} -1/2 & -1 & 1/2 \\ 1 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1/2 & -1 & 1/2 \\ 1 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{pmatrix} = \begin{pmatrix} -1/2 & -2 & 1/2 \\ -1 & -1 & 1 \\ -1/2 & -1 & 3/2 \end{pmatrix}$$

COLEGIO ALMA'S
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APELLIDOS Y NOMBRE: ....

CURSO: .... **Nº** ....

FECHA: .... **ASIGNATURA:** ....

6. $r \equiv \begin{cases} x+y+z=1 \\ x-2y-2z=0 \end{cases}$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & -2 & -2 \end{vmatrix} = 3\vec{j}-3\vec{k} \longrightarrow \vec{v}(0,1,-1)$.
P(

$\pi \equiv 2x+y+mz-3=0 \longrightarrow \vec{n}(2,1,m)$

$\vec{n}\cdot\vec{v} = 0+1-m=0 \rightarrow m=1$ → Si $m \in \mathbb{R}-\{1\}$ son secantes.
→ Si $m=1$ son paralelos ya que no comparten ningún pto.

$P_\pi(0,3,0) \longrightarrow \left.\begin{matrix} 0+3+0 \neq 1 \\ 0-6+0 \neq 0 \end{matrix}\right\}$

7. $\begin{vmatrix} 1 & -1 & 2 & 3 \\ 2 & 1 & 0 & 1 \\ 3 & -1 & 1 & 2 \\ 2 & -1 & 0 & 1 \end{vmatrix} \overset{F_1-2F_3}{=} \begin{vmatrix} -5 & 1 & 0 & -1 \\ 2 & 1 & 0 & 1 \\ 3 & -1 & 1 & 2 \\ 2 & -1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} -5 & 1 & -1 \\ 2 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} \overset{F_2-F_3}{} \begin{vmatrix} -5 & 1 & -1 \\ 0 & 2 & 0 \\ 2 & -1 & 1 \end{vmatrix}$

$= 2\begin{vmatrix} -5 & -1 \\ 2 & 1 \end{vmatrix} = 2(-5+2) = \boxed{-6}$

8. $P(1,-2,1)$, $Q(-4,0,1)$, $R(-3,1,2)$, $S(0,-3,0)$

$\vec{u}(5,-2,0)$, $\vec{v}(4,-3,-1)$, $\vec{w}(3,-4,-2)$

a) $\begin{vmatrix} x-1 & y+2 & z-1 \\ 5 & -2 & 0 \\ 4 & -3 & -1 \end{vmatrix} = 0$

$2(x-1)+5(y+2)-7(z-1)=0$

$\boxed{2x+5y-7z+15=0}$

b) $\dfrac{x-1}{5} = \dfrac{y+2}{-2} = \dfrac{z-1}{0} \longrightarrow \left.\begin{matrix} z-1=0 \\ 2x+5y+8=0 \end{matrix}\right\}$ Recta PQ

$\left.\begin{matrix} x=3\lambda \\ y=-3-4\lambda \\ z=-2\lambda \end{matrix}\right\}$ Recta RS

$\vec{u} \not\parallel \vec{w} \rightarrow$ Son secantes o se cruzan.

$\left.\begin{matrix} -2\lambda-1=0 \rightarrow \lambda=-1/2 \\ 6\lambda-15-20\lambda-16\lambda=0 \rightarrow -30\lambda=15 \rightarrow \lambda=-1/2 \end{matrix}\right\}$ Se cortan en 1 pto

$P(-3/2,-1,1)$

COLEGIO ALMA'S
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APELLIDOS Y NOMBRE:

CURSO: Nº

FECHA: ASIGNATURA:

9.- $\begin{vmatrix} 2a & 6 & x+4 \\ 2b & 9 & y+6 \\ 2c & -3 & z-2 \end{vmatrix} = 6 \begin{vmatrix} a & 2 & x+4 \\ b & 3 & y+6 \\ c & -1 & z-2 \end{vmatrix} = 6 \begin{vmatrix} a & 2 & x \\ b & 3 & y \\ c & -1 & z \end{vmatrix}$ = Trasponemos

$C_1/2 \quad C_2/3 \qquad C_3 - 2C_2$

$= 6 \begin{vmatrix} a & b & c \\ 2 & 3 & -1 \\ x & y & z \end{vmatrix} = 6 \cdot 2 = \boxed{12}$

10.- $\left.\begin{array}{l} \pi_1 \equiv ax + y - z + 1 = 0 \\ \pi_2 \equiv x + ay + z - 2 = 0 \end{array}\right\} \begin{array}{l} \vec{n}_1 (a, 1, -1) \\ \vec{n}_2 (1, a, 1) \end{array}$

a) Para que sean paralelos $\frac{a}{1} = \frac{1}{a} = \frac{-1}{1} \Rightarrow \boxed{a = -1}$

b) Para que sean perpendiculares $\vec{n}_1 \cdot \vec{n}_2 = a + a - 1 = 0 \Rightarrow \boxed{a = 1/2}$