

Corrección Recuperación - 2ª Evaluación - Matemáticas 2 06-04-18

2.  $A = \begin{pmatrix} 1 & -4 \\ -1 & 3 \end{pmatrix}$   $|A| = 3 - 4 = -1 \neq 0 \Rightarrow \exists A^{-1}$   $AdA = \begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix}$   $A^{-1} = \begin{pmatrix} -3 & -4 \\ -1 & -1 \end{pmatrix}$

a)  $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$   $|B| = 1 - 1 = 0 \Rightarrow \nexists B^{-1}$

b)  $X = A^{-1}(2B + I) = \begin{pmatrix} -3 & -4 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -6 \\ -1 & -1 \end{pmatrix}$

2.  $P(2, 3, 4)$   
a)  $\pi \equiv x + y + 2z + 4 = 0$   $\rightarrow \bar{r}(1, 1, 2)$   $\rightarrow \frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{2}$

b)  $r = \begin{cases} x = 1 + \lambda \\ y = 2 + \lambda \\ z = 2 + 2\lambda \end{cases}$   $s = \begin{cases} 2x - 4y = 2 - 2a \\ 3y - 2z = 2 \end{cases}$

$\bar{U}(1, 1, 2)$   $\bar{U}(a, 2, 3) \rightarrow \bar{D} \times \bar{U} \rightarrow$  se cortan o no.

$2(1 + \lambda) - 4(2 + \lambda) = 2 - 2a \rightarrow 2 - 2a = 2 - 2a \Rightarrow 0a = 0$

$3(2 + \lambda) - 2(2 + 2\lambda) = 2 \rightarrow 6 + 3\lambda - 4 - 4\lambda = 2 \rightarrow \lambda = 0$

Es decir, las rectas se cortan para cualquier valor de  $a$

3. Matriz Asociada Matriz Ampliada

a)  $A = \begin{pmatrix} -1 & a & 2 \\ 2 & a & -1 \\ a & -1 & 2 \end{pmatrix}$   $A^* = \begin{pmatrix} -1 & a & 2 & a \\ 2 & a & -1 & 2 \\ a & -1 & 2 & a \end{pmatrix}$

$|A| = (-2a - 4 - a^2) - (2a^2 - 1 + 4a) = -3(a+1)^2 = 0 \Rightarrow a = -1$

Si  $a \in \mathbb{R} - \{-1\} \Rightarrow \text{rang} A = \text{rang} A^* = 3 \Rightarrow$  Sistema Compatible Determinado

Si  $a = -1 \Rightarrow \text{rang} A = \text{rang} A^* = 2 \Rightarrow$  Sistema Compatible Indeterminado

$A^* = \begin{pmatrix} -1 & -1 & 2 & -1 \\ 2 & -1 & -1 & 2 \\ -1 & -1 & 2 & -1 \end{pmatrix}$   $F_3 = F_1 \Rightarrow \text{rang} A^* \neq 3$   $\begin{vmatrix} -1 & -1 \\ 2 & -1 \end{vmatrix} = 1 + 2 = 3 \neq 0$

Si  $a=2 \Rightarrow |A| = -3(2+1)^2 = -27$

$$x = \frac{\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & 1 & 2 \end{vmatrix}}{-27} = \boxed{\frac{2}{3}}$$

$$y = \frac{\begin{vmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & 2 & 2 \end{vmatrix}}{-27} = \boxed{\frac{2}{3}}$$

$$z = \frac{\begin{vmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & -1 & 2 \end{vmatrix}}{-27} = \boxed{\frac{2}{3}}$$

4.  $\left. \begin{matrix} \vec{D}_1(2,1,2) \\ \vec{D}_2(2,3,1) \end{matrix} \right\} \Rightarrow \vec{D}_1 \wedge \vec{D}_2$  se cortan o se cruzan  $r = \begin{cases} x=2\lambda \\ y=\lambda \\ z=1+2\lambda \end{cases}$

a)  $S = \begin{cases} 3x=2y-2 \\ y-1=3z \end{cases} \rightarrow \begin{cases} 6\lambda=2\lambda-2 \Rightarrow \lambda_1=-1/2 \\ y-1=3+6\lambda \rightarrow \lambda_2=-4/5 \end{cases} \left. \begin{matrix} \lambda_1 \neq \lambda_2 \\ \Rightarrow \text{se cruzan.} \end{matrix} \right\}$

b)  $P(0,0,0)$

Plano que contiene a  $r$  y pasa por  $P$

$$\begin{vmatrix} x & y & z \\ 2 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 0 \rightarrow x-2y=0$$

Plano que contiene a  $S$  y pasa por  $P$

$$\begin{vmatrix} x & y & z \\ 2 & 3 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0 \rightarrow x-2z=0$$

la recta sería:

$$\boxed{\begin{cases} x-2y=0 \\ x-2z=0 \end{cases}}$$

5.  $|A| = m^3 + 2m^2 - m - 2 = (m+1)(m-1)(m+2)$

a) Si  $m \in \mathbb{R} - \{-1, 1, -2\} \Rightarrow |A| \neq 0 \Rightarrow \exists A^{-1}$

b)  $m=0 \Rightarrow |A| = -2$   $\text{Adj}A = \begin{pmatrix} -1 & -2 & -1 \\ 0 & -2 & -2 \\ -1 & 0 & 2 \end{pmatrix}$   $(\text{Adj}A)^t = \begin{pmatrix} -1 & 0 & -1 \\ -2 & -2 & 0 \\ -1 & -2 & 2 \end{pmatrix}$

$$\boxed{A^{-1} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1 & 1 & 0 \\ 1/2 & 1 & -1 \end{pmatrix}}$$

6.  $\left. \begin{matrix} \vec{n}_\pi(1,-1,-1) \\ \vec{D}_r(1,2,-4) \end{matrix} \right\} \vec{D} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ 1 & 2 & -4 \end{vmatrix} = 6\vec{i} + 3\vec{j} + 3\vec{k} \rightarrow \vec{D}(2,1,1)$

$$\boxed{\frac{x-2}{2} = y+1 = z+2}$$

$$\textcircled{7.} \begin{vmatrix} 1 & 2 & -2 \\ 3 & -1 & 1 \\ 2 & -1 & -4 \\ 4 & -3 & 1 \end{vmatrix} \begin{array}{l} E_2 - 3E_1 \\ E_3 - 2E_1 \\ E_4 - 4E_1 \end{array} = \begin{vmatrix} 1 & 2 & 1 & -2 \\ 0 & -7 & -4 & 7 \\ 0 & -5 & 0 & 0 \\ 0 & -11 & -6 & 9 \end{vmatrix} = \begin{vmatrix} -7 & -4 & 7 \\ -5 & 0 & 0 \\ -11 & -6 & 9 \end{vmatrix} = 5 \begin{vmatrix} -4 & 7 \\ -6 & 9 \end{vmatrix} = 5(-36 + 42) = 5 \cdot 6 = \boxed{30}$$

$$\textcircled{8.} \begin{array}{l} P(0,0,0) \\ Q(1,1,1) \\ R(3,0,0) \end{array} \left. \begin{array}{l} \overline{PQ}(1,1,1) \\ \overline{PR}(1,0,0) \end{array} \right\} \begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} y & z \\ 1 & 1 \end{vmatrix} \rightarrow \boxed{y-z=0}$$

b) Para que sean coplanarios  $S(0,3,0)$  debe pertenecer a  $y-z=0$   
Si sustituimos  $3-0 \neq 0 \Rightarrow$  No son coplanarios

$$\textcircled{9.} A \rightarrow \begin{vmatrix} 2a & 2b & 2c \\ 2x+a & 2y+b & 2z-c \\ 1 & 1 & 1 \end{vmatrix} \begin{array}{l} E_1/2 \\ E_2 - E_1 \end{array} = 2 \begin{vmatrix} a & b & c \\ 2x+a & 2y+b & 2z-c \\ 1 & 1 & 1 \end{vmatrix} \begin{array}{l} E_2 - E_1 \\ E_2/2 \end{array} = 2 \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ 1 & 1 & 1 \end{vmatrix} \begin{array}{l} E_1/2 \\ E_2/2 \end{array} = 4 \begin{vmatrix} a & b & c \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = 4 \cdot 5 = \boxed{20}$$

$$\textcircled{10.} r = \begin{cases} x=0 \\ y=0 \end{cases} \left. \begin{array}{l} \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0\bar{i} + 0\bar{j} + 1\bar{k} \rightarrow \vec{v}(0,0,1) \\ P(0,0,0) \end{array} \right\} \begin{array}{l} x=0 \\ y=0 \\ z=\lambda \end{array}$$

$$0-0 + a\lambda = 1 \rightarrow a\lambda = 1 \rightarrow \begin{array}{l} \text{Si } a=0 \Rightarrow \text{Paralelas} \\ \text{Si } a \neq 0 \Rightarrow \text{Secantes} \end{array}$$